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**MCM**
**The Relationship Between Confidence and Clearance**

Consider the problem of $m$ mines remaining in a minefield, while $n$ have been found in clearing operations so far. Thus, $m+n$ represents the total number of mines. The probability that a mine is found is $p$. The success of another mine being found may be thought as a discrete event in a series of Bernoulli trials and thus can be described by the following instance of the negative binomial distribution.

1. $P(m) = \binom{m+n}{m} p^{n+1}(1-p)^m$

Consider the special case where no mines have been found. In this situation $n = 0$ and $\binom{m+n}{n} = 1$. Thus, statement (1) reduces to:

2. $P(m) = p(1-p)^m$

We will apply statement (2) to a common example in MCM, where a field has been cleared to 64% during exploratory operations. First, we will consider what the probability of $m = 0$, indicating 0 mines left in the field. Then we will consider $m = 1$ and $m = 2$, the scenarios where 1 mine and 2 mines remain in the field respectively.

3. $P(0) = .64(.36)^0 = .64$

4. $P(1) = .64(.36)^1 = .23$

5. $P(2) = .64(.36)^2 = .08$

The combined probability of statements (3), (4) and (5) is .95. This will be familiar to MCM personnel as one method to derive the common assertion that 64% clearance gives 95% confidence that 0, 1 or 2 mines remain in the field, assuming no mines have been found thus far.

From the preceding statements we can generalize the probability of finding greater than $t$ mines as:

6. $S(m) = \sum_{m=t}^{n} p(1-p)^m = p(1-p)^t + p(1-p)^{t+1} + p(1-p)^{t+2} + p(1-p)^{t+3} + \ldots + p(1-p)^n$

Note we can factor out product constants from summations. Thus,

$p * \sum_{m=t}^{n}(1-p)^m = p((1-p)^t + (1-p)^{t+1} + (1-p)^{t+2} + (1-p)^{t+3} + \ldots + (1-p)^n)$

This simplification allows the application of a known and proven identity:

$\sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}$

Thus, statement (6) may be reduced to:

7. $S(m) = p * \frac{1-(1-p)^{n+1}}{1-(1-p)}$

Simplifying with algebra:

8. $S(m) = p * \frac{1-(1-p)^{n+1}}{1-1+p}$

9. $S(m) = p * \frac{1-(1-p)^{n+1}}{p}$

10. $S(m) = 1-(1-p)^{n+1}$

11. $S(m) - 1)^{1/n+1} = (1-p)$

12. $1-(1-S(m))^{1/n+1}=p$

Equation (12) is more often depicted in MCM circles as:

A. $1-(1-CL)^{1/t+1}=P$

Note that in statement (6), S(m) (referred to as CL in equation (A)), represents the combined probability that 0 to $t$ mines exist in the field. This is typically referred to as confidence level. In this context confidence may be thought as the probability of success from a series of events when only the total number of events and successes are known. In practical terms, confidence level is the expected likelihood that there are a given number of mines left in the field. In this statement $P$ refers to the clearance achieved in the minefield. Therefore, equation (A) describes the relationship between clearance achieved and confidence level. It is important to stress that this equation only holds true if no mines are found. As an example, we will verify the example used in statements 3-5. In this example, $t$ = 2, and P = .64.

13. $1-(1-CL)^{1/3}=.64$

14. $(1-CL)^{1/3}=.36$

15. $(1-CL)=.36^3$

16. $CL=.953$

Thus, we have demonstrated again that when a field is cleared to 64% we have 95% confidence that $t \in \{0,1,2\}$, assuming no mines have been found, using equation (A). By changing the variables we can then calculate confidence levels for any achieved level of clearance and desired $t$. This confidence may then be used in MIW operations to make risk decisions. Alternatively, we can also determine the needed clearance for a desired confidence level for a given $t$. This information can be used during planning to determine the required extent of mine sweeping and mine hunting operations.

For the next example we will calculate the confidence that $t \in \{0,1\}$ when a field has been cleared to 95%.

17. $1-(1-CL)^{1/2}=.95$

18. $(1-CL)^{1/2}=.05$

19. $CL=1-.05^2$

20. $CL=.997$

We determine that 95% clearance gives us 99% confidence that there are 0 or 1 mines left in the field. This further illustrates the use of this equation. Additionally, it provides an important insight into MCM operations. To increase confidence from 95% that $t \in \{0,1,2\}$ to 99% that $t \in \{0,1\}$ a substantial amount of additional clearance must be achieved. This type of result could provide evidence towards the infeasibility of attaining a desired confidence given operational time constraints, environmental conditions and available assets.

For example, clearance operations against an acoustic mine in a poor acoustic environment may severely reduce the associated $A$ and $B$ values of the minesweeping device. This will then require a large number of runs over smaller tracks with the device, which may not be possible within the allotted time. This problem may be further exacerbated by the existence of any mine counter counter-measures. Therefore, planners may determine that only 80% clearance is achievable given constraints. Using (A) we can calculate that this clearance provides 96% confidence that $t \in \{0, 1\}$. The operational MCM Commander may then decide that this reduced confidence is still sufficiently low to allow transitors through those waters.

As a final example we will determine the needed clearance to achieve 95% confidence that $t \in \{0, 1, 2, 3, 4\}$.

21. $1 - (1 - .95)^{1/5} = P$

22. $1 - .05^{1/5} = P$

23. $.4507 = P$

At 45% clearance, assuming no mines are found, we may have 95% confidence that $t \in \{0, 1, 2, 3, 4\}$. This example illustrates that if acceptable values for $t$ are increased, required clearance levels are drastically reduced, which requires less time, and fewer assets. However, proportional increases in risk to transitors must be considered.